Introduction to the Standard Model
William and Mary PHYS 771 Spring 2014

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Class information, including syllabus and homework assignments can be found at
`http://ntc0.lbl.gov/~walkloud/wm/courses/PHYS_771/`
or
`http://cyclades.physics.wm.edu/~walkloud/wm/PHYS_771/`

Homework Assignment 3: due Friday 28 March

1. Consider two fermions (e, μ) and a complex scalar coupled through $U(1)$ gauge symmetry

$$\mathcal{L} = \sum_{f=e,\mu} \bar{\psi}_f \left[i \not{D} - m_f \right] \psi_f + [D_\mu \phi]^\dagger [D^\mu \phi] - m_\phi^2 \phi^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{1}$$

with $D_\mu = \partial_\mu + iqA_\mu$.

(a) compute the unpolarized (spin-averaged) differential cross section for $e^+e^- \rightarrow \mu^+\mu^-$ in the limit $m_e = 0$ in the center of mass frame

$$\frac{d\sigma}{d\cos\theta} =? \tag{2}$$

(b) same as 1a but for $e^+e^- \rightarrow \phi^+\phi^-$

(c) compare these cross sections in the relativistic limit (ignore masses of final states as well)

2. Pion decay: the pion decay amplitude is proportional to

$$M_{\pi^- \to \ell\nu_\ell} \propto f_\pi q^\mu \bar{u}_\ell \gamma_\mu (1 - \gamma_5) v_{\bar{\nu}_\ell} \,. \tag{3}$$

Recalling the $(A \rightarrow 1 + 2)$ decay rate is given in the rest frame of A by

$$\Gamma = \frac{|\vec{p}_f|}{32\pi^2 M_A^2} \int d\Omega |M|^2 \tag{4}$$

(a) calculate the ratio

$$R = \frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} \tag{5}$$

and compare with the PDG value

(b) suppose the weak decay amplitude were instead given by

$$\tilde{M}_{\pi^- \to \ell\nu_\ell} \propto g_\pi \bar{u}_\ell \gamma_5 v_{\bar{\nu}_\ell} \,, \tag{6}$$

compute the corresponding ratio R for this pseudo-scalar model

3. In class, we discussed the complex ϕ^4 theory with spontaneous symmetry breaking. We worked in 'polar' coordinates with $\phi = \frac{\rho+v}{\sqrt{2}}e^{i\theta}$. We computed the one-loop s-channel contribution to $\theta\theta \to \theta\theta$ scattering amplitude with intermediate ρ particles, using dimensional regularization, finding

$$\delta A(s) = \frac{1}{2}\left(\frac{s}{4\pi v^2}\right)^2 \left[\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) - \ln\frac{m^2}{\mu^2} - 2 + \begin{cases} i\sigma(s)(2\tan^{-1}(i\sigma(s)) - \pi) & s < 4m^2 \\ \sigma(s)\ln\frac{1+\sigma(s)}{1-\sigma(s)} - i\pi\sigma(s) & s > 4m^2 \end{cases}\right. \tag{7}$$

with $\sigma(s) = \sqrt{1 - 4m^2/s}$.

   (a) Compute the t and u channel contributions to this scattering amplitude and combine them with this s-channel result

   (b) to renormalize the theory, we must add a higher dimensional operator, with four derivatives and four θ fields. Write down the form of this operator.

   (c) use this new operator to act as the counterterm to renormalize the scattering amplitude. What is the $1/\epsilon$ contribution to this operator?

   (d) by requiring the renormalized amplitude to be independent of the dimensional-regularization scale μ, determine the μ dependence of the coefficient of the new operator

4. Suppose we have a charge distribution $\rho(r) = Ne^{-mr}$ with $\int d^3r\rho(r) = 1$.

   (a) what is $N =$?

   (b) what is the resulting form factor for Compton scattering

$$F(\vec{q}) = \int d^3r e^{i\vec{q}\cdot\vec{r}}\rho(\vec{r}) = ? \tag{8}$$

   (c) Given the Taylor expansion of the form factor

$$F(q) = 1 - \frac{1}{6}r^2q^2 + \ldots \tag{9}$$

   What is the predicted 'charge-radius' of the proton, using the electric form factor, $G_E(-q^2) = (1 - q^2/m_0^2)^{-2}$ with $m_0^2 = 0.71$ GeV^2? Compare with the PDG value of the charge radius and the recent determination from muonic-Hydrogen (use Google to find these values).